

# QCD predictions for annihilation decays of P-wave quarkonia to next-to-leading order in $\alpha_s$

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## Abstract

The decay rates of P-wave heavy quarkonia to light hadrons are presented to leading order in  $v^2$  and next-to-leading order in  $\alpha_s$ . They include contributions from both the color-singlet component and the color-octet component of quarkonia. Applying these results to charmonium and using measured decay rates for the  $\chi_{c1}$  and  $\chi_{c2}$  by E760, we determine the two nonperturbative decay matrix elements, and then predict the hadronic decay rates of  $\chi_{c0}$  and  $h_c$ , and the electromagnetic decay rates of  $\chi_{c0}$  and  $\chi_{c2}$ . The obtained decay rates of  $\chi_{c0} \rightarrow LH$  and  $\chi_{c0} \rightarrow \gamma\gamma$  are in agreement with the Crystal Ball result, and also with the new measurement by BES. However, the results for  $\Gamma(\chi_{c0} \rightarrow LH)$  are dependent on the choice of renormalization scale.

The study of heavy quarkonium physics can provide very interesting tests of perturbative quantum chromodynamics (PQCD). Calculations of the rates for heavy quarkonium decay into light hadrons were among the early applications of PQCD. These early calculations are based on a naive factorization assumption that all long-distance nonperturbative effects can be factored into the nonrelativistic wavefunction of color singlet  $Q\bar{Q}$  or its derivative at the origin, and the perturbative part is related to the annihilation rates of color-singlet  $Q\bar{Q}$  which can be calculated using PQCD. In the nonrelativistic limit, this early factorization formalism was supported by explicit calculations for S-wave decays at next-to-leading order in  $\alpha_s$  [1]. But in the case of P-wave [2] quarkonium decays, infrared divergences appeared in the perturbative calculations of color-singlet  $Q\bar{Q}$  annihilation amplitudes. These are clear indications that the decay rates are sensitive to nonperturbative effects beyond those related to the wavefunction of color-singlet  $Q\bar{Q}$  pair or its derivative at the origin, and not all nonperturbative effects can be factored into the color-singlet component of quarkonium. Recently, Bodwin, Braaten and Lepage (BBL) have developed a rigorous factorization formalism [3] which is based on an effective field theory, nonrelativistic QCD (NRQCD). This factorization formalism provides a clean separation between short-distance effects and long-distance effects for the decay rates and production cross sections of heavy quarkonium.

Nowadays there is a renewed interest in studying the decay of P-wave charmonium, not only due to the theoretical development mentioned above but also due to recent experimental results such as the total decay widths of  $\chi_{cJ}$  and the observation of  $h_c$ . BBL have applied the new factorization approach in a phenomenological analysis of P-wave charmonium decays [4]. They give a leading order result with both the color-singlet and color-octet  $Q\bar{Q}$  components. The next-to-leading order correction to the decay of  $h_c$  is given in [5], where both the color-singlet and color-octet contributions are included and the explicit cancellation of previously encountered infrared divergence is revealed. In [6] the next-to-leading order color-singlet terms are considered in a phenomenological analysis of hadronic annihilation decays of  $\chi_{cJ}$ . Recently the next-to-leading order color-octet corrections to hadronic  $\chi_J$  decays have also been calculated [7]. In this paper we will perform a phenomenological study for the hadronic decays of four P-wave charmonium states by using the results that completely include the next-to-leading order QCD corrections.

We start with the formulas for the P-wave quarkonium decay widths in the new factorization formalism

$$\Gamma(\chi_J \rightarrow LH) = 2\text{Im}f_1(^3P_J)H_1 + 2\text{Im}f_8(^3S_1)H_8 + O(v^2\Gamma), \quad (1)$$

$$\Gamma(h \rightarrow LH) = 2\text{Im}f_1(^1P_1)H_1 + 2\text{Im}f_8(^1S_0)H_8 + O(v^2\Gamma), \quad (2)$$

where  $H_1$  and  $H_8$  are the matrix elements of color-singlet and color-octet operators respectively. The short-distance coefficients can be extracted by matching the imaginary part of the on-shell  $Q\bar{Q}$  pair forward

scattering amplitude calculated in full perturbative QCD with that calculated in NRQCD. We list the results to next-to-leading order in  $\alpha_s$  as the following

$$\begin{aligned} Imf_1(^3P_0) &= (Imf_1(^3P_0))_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left[ (4b_0 - \frac{4n_f}{27}) \ln \frac{\mu}{2m} \right. \right. \\ &\quad \left. \left. + \left( \frac{454}{81} - \frac{\pi^2}{144} \right) C_A + \left( -\frac{7}{3} + \frac{\pi^2}{4} \right) C_F - \frac{58}{81} n_f \right] \right\}, \end{aligned} \quad (3)$$

$$Imf_1(^3P_1) = (Imf_1(^3P_0))_0 \frac{\alpha_s}{\pi} \left[ -\frac{4n_f}{27} \ln \frac{\mu}{2m} + \left( \frac{587}{54} - \frac{317\pi^2}{288} \right) - \frac{16n_f}{81} \right], \quad (4)$$

$$\begin{aligned} Imf_1(^3P_2) &= (Imf_1(^3P_2))_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left[ (4b_0 - \frac{5n_f}{9}) \ln \frac{\mu}{2m} \right. \right. \\ &\quad \left. \left. + \left( \frac{2239}{216} - \frac{337\pi^2}{384} + \frac{5\ln 2}{3} \right) C_A - 4C_F - \frac{29}{27} n_f \right] \right\}, \end{aligned} \quad (5)$$

$$Imf_1(^1P_1) = \frac{(N_c^2 - 4)C_F\alpha_s^3}{3N_c^2} \left( \frac{7\pi^2 - 118}{48} - \ln \frac{\mu}{2m} \right), \quad (6)$$

$$\begin{aligned} Imf_8(^3S_1) &= (Imf_8(^3S_1))_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left[ 4b_0 \ln \frac{\mu}{2m} - \frac{5}{9} n_f \right. \right. \\ &\quad \left. \left. + \left( \frac{133}{18} + \frac{2}{3} \ln 2 - \frac{\pi^2}{4} \right) C_A - \frac{13}{4} C_F + \frac{5}{n_f} \left( -\frac{73}{4} + \frac{67}{36} \pi^2 \right) \right] \right\}, \end{aligned} \quad (7)$$

$$Imf_8(^1S_0) = (Imf_8(^1S_0))_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left[ 4b_0 \ln \frac{\mu}{2m} - \frac{8}{9} n_f + \left( \frac{\pi^2}{4} - 5 \right) C_F + \left( \frac{479}{36} - \frac{17\pi^2}{24} \right) C_A \right] \right\} \quad (8)$$

, where

$$b_0 = \frac{1}{12} (11C_A - 2n_f),$$

and  $C_F = \frac{N_c^2 - 1}{2N_c}$ ,  $C_A = N_c$ . The coefficients  $Imf_1(^3P_1)$  starts in order  $\alpha_s^3$ , hence they only are given to leading order; while all other coefficients start in order  $\alpha_s^2$ , whose next-to-leading order corrections are also given. We note that (6) and (8) are given in [5] and (7) is given in [7]. Coefficients  $Imf_1(^3P_J)$  in (3)-(5) have been calculated in [2] and listed in [7], where quark and antiquark are taken off-shell and binding energy regularization scheme was used. Here we recalculate them by using dimensional regularization to control the infrared divergence, and there are some differences between our results (3)-(5) and that in [7]. This difference only comes from the diagram in Fig.1 which represents the inclusive processes  $Q\bar{Q}(^3P_J) \rightarrow q_i\bar{q}_ig$ . In the  $d = 4 - 2\epsilon$  dimension space, contributions of the three particle cut diagram in Fig.1 to the imaginary

part of  $Q\bar{Q}(^3P_J)$  pair scattering amplitude are

$$Im\mathcal{M}_{Fig.1}^{full\ QCD}(Q\bar{Q}(^3P_0)) = (Imf_8(^3S_1))_0 f(\epsilon) \frac{4C_F\alpha_s}{3N_C\pi} \left(-\frac{1}{2\epsilon_{IR}}\right) + (Imf_1(^3P_0))_0 \frac{\alpha_s}{\pi} \left(-\frac{58n_f}{81} - \sum_i \frac{2}{3} \ln \frac{m_i}{2m}\right), \quad (9)$$

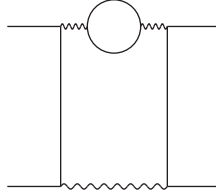
$$Im\mathcal{M}_{Fig.1}^{full\ QCD}(Q\bar{Q}(^3P_1)) = (Imf_8(^3S_1))_0 f(\epsilon) \frac{4C_F\alpha_s}{3N_C\pi} \left(-\frac{1}{2\epsilon_{IR}}\right) + (Imf_1(^3P_0))_0 \frac{\alpha_s}{\pi} \left(-\frac{16n_f}{81}\right), \quad (10)$$

$$Im\mathcal{M}_{Fig.1}^{full\ QCD}(Q\bar{Q}(^3P_2)) = (Imf_8(^3S_1))_0 f(\epsilon) \frac{4C_F\alpha_s}{3N_C\pi} \left(-\frac{1}{2\epsilon_{IR}}\right) + (Imf_1(^3P_2))_0 \frac{\alpha_s}{\pi} \left(-\frac{29n_f}{27} - \sum_i \frac{2}{3} \ln \frac{m_i}{2m}\right). \quad (11)$$

Here

$$f(\epsilon) = \left(\frac{4\pi\mu^2}{4m^2}\right)^\epsilon \Gamma(1+\epsilon)$$

The results coming from the diagrams that represent the inclusive processes  $Q\bar{Q}(^3P_J) \rightarrow gg$  and  $Q\bar{Q}(^3P_J) \rightarrow ggg$  are finite and have been given in [2].



**Fig.1 Feynman diagram with three particle cut contributing to the divergence terms in the full theory calculation of  $Q\bar{Q}$  annihilation amplitudes**

While in the effective field theory NRQCD, the corresponding scattering amplitudes can be written as

$$Im\mathcal{M}(^3P_J)_{NRQCD} = \frac{Imf_1(^3P_J)}{m^6} + (Imf_8(^3S_1))_0 \frac{4C_F\alpha_s}{3m^6 N_C\pi} \left(-\frac{1}{2\epsilon_{IR}} + \frac{1}{2\epsilon_{UV}}\right). \quad (12)$$

Comparing (12) with (9)–(11), it is obvious that the divergence terms are removed and finite coefficients  $Imf_1(^3P_J)$  (3)–(5) can be obtained. It is important to point out that if one replaces  $\ln \frac{m}{\epsilon}$  in the expressions in [7] by  $-\frac{1}{2\epsilon_{IR}}$ , then the divergent terms are the same as those in (9)–(11). The difference only occurs in their finite terms due to different regularization scheme being used. It is certainly true that the coefficients of 4-fermion operators must be infrared finite and independent of the choice of regularization procedures because all nonperturbative effects are factored into the matrix elements. Note that the coefficients can be

derived consistently only by taking the same regularization scheme in full QCD and in effective NRQCD. The advantage of using dimensional regularization is that the on-shell condition and gauge invariance are maintained manifestly and conventional treatment of NRQCD is under the on-shell condition, thus we can give an explicit cancellation for divergences appeared previously. The introduction of off-shell binding energy makes it difficult to do calculation in NRQCD, and the results are incomplete if simply absorbing the divergences associated with the logarithm of binding energy into the matrix elements of color-octet operators.

Now we apply the factorization formula to charmonium systems. For the lowest radial excitation, the  $^3P_J$  states are called  $\chi_{cJ}$  and the  $^1P_1$  state is called  $h_c$ . The explicit form for their decay rates into light hadrons at leading order in  $v^2$  are

$$\Gamma(\chi_{c0} \rightarrow LH) = C_{00}\alpha_s^2(m_c)(1 + C_{01}\frac{\alpha_s}{\pi})H_1 + D_0\alpha_s^2(m_c)(1 + D_1\frac{\alpha_s}{\pi})H_8(m_c), \quad (13)$$

$$\Gamma(\chi_{c1} \rightarrow LH) = C_1\alpha_s^3H_1 + D_0\alpha_s^2(m_c)(1 + D_1\frac{\alpha_s}{\pi})H_8(m_c), \quad (14)$$

$$\Gamma(\chi_{c2} \rightarrow LH) = C_{20}\alpha_s^2(m_c)(1 + C_{21}\frac{\alpha_s}{\pi})H_1 + D_0\alpha_s^2(m_c)(1 + D_1\frac{\alpha_s}{\pi})H_8(m_c), \quad (15)$$

$$\Gamma(h_c \rightarrow LH) = C'_1\alpha_s^3H_1 + D'_0\alpha_s^2(m_c)(1 + D'_1\frac{\alpha_s}{\pi})H_8(m_c), \quad (16)$$

where “LH” on the left hand of (13)—(16) represents all final states consisting of light hadrons, and the coefficients are

$$C_{00} = \frac{4\pi}{3}, \quad C_{01} = 8.710;$$

$$C_1 = -0.370, \quad C'_1 = -0.161;$$

$$C_{20} = \frac{16\pi}{45}, \quad C_{21} = -5.061;$$

$$D_0 = \pi, \quad D_1 = 4.110;$$

$$D'_0 = \frac{5\pi}{6}, \quad D'_1 = 6.66.$$

In deriving these coefficients we have taken  $N_c = 3$ ,  $n_f = 3$  and made a choice  $\mu = m_c$  for the scale in the  $\overline{MS}$  scheme. The large size of some coefficients for the correction terms is apparent. These numbers obviously depend on the definition of the renormalized couplings  $\alpha_s$ . We will study the the renormalization scale dependence of the results later. In the following we use measured decay rates of the  $\chi_{c1}$  and  $\chi_{c2}$  to predict the inclusive decay rates of the  $\chi_{c0}$  and  $h_c$ , and the theoretical uncertainties will be estimated by considering relativistic corrections and high order perturbative QCD corrections.

Precision measurements of the total decay rates of the  $^3P_1$  state  $\chi_{c1}$  and  $^3P_2$  state  $\chi_{c2}$  have recently been carried out at Fermilab by the E760 collaboration. Their results with statistical and systematic errors are [8]

$$\Gamma(\chi_{c2}) = 2.00 \pm 0.18 \text{ Mev},$$

$$\Gamma(\chi_{c1}) = 0.88 \pm 0.14 \text{ Mev}.$$

It is well known that the main decay modes of these P-wave charmonium states are the decay into light hadrons and the radiative transitions into  $J/\psi$  or  $\eta_c$ . Other decay modes such as pionic transitions of the P states to the S states, of which the most important decay modes should be  $J/\psi + \pi\pi$  and  $\eta_c + \pi\pi$ , contribute much less to the total widths and therefore can be neglected [9]. Previous experiments have measured the branching fractions for the radiative transitions of the  $\chi_{c1}$  and  $\chi_{c2}$  into the  $J/\psi$ , and they are  $B(\chi_{c1} \rightarrow \gamma J/\psi) = 0.273 \pm 0.016$ , and  $B(\chi_{c2} \rightarrow \gamma J/\psi) = 0.135 \pm 0.011$  [8]. We use the radiative branching fractions and the total decay rates to obtain the partial rates for light hadronic decays of the  $\chi_{c1}$  and  $\chi_{c2}$

$$\Gamma(\chi_{c1} \rightarrow LH) = 0.64 \pm 0.10 \text{ Mev}, \quad (17)$$

$$\Gamma(\chi_{c2} \rightarrow LH) = 1.71 \pm 0.16 \text{ Mev}. \quad (18)$$

$H_1$  and  $H_8$  can be obtained directly by using (14) and (15),

$$H_1 = \frac{\Gamma(\chi_{c2} \rightarrow LH) - \Gamma(\chi_{c1} \rightarrow LH)}{C_{20}\alpha_s^2(1 + C_{21}\frac{\alpha_s}{\pi}) - C_1\alpha_s^3}, \quad (19)$$

$$H_8 = \frac{\Gamma(\chi_{c2} \rightarrow LH) - C_{20}\alpha_s^2(1 + C_{21}\frac{\alpha_s}{\pi})H_1}{D_0\alpha_s^2(1 + D_1\frac{\alpha_s}{\pi})}. \quad (20)$$

Here we determine  $\alpha_s(m_c)$  by taking the coupling constant  $\alpha_s(m_b) = 0.189 \pm 0.008$  extracted from bottomonium decays and evolving it down to the scale  $m_c$ . The resulting value of the coupling constant is  $\alpha_s(m_c) = 0.29 \pm 0.02$ . Inserting (17) and (18) into (19) and (20), we obtain

$$H_1 = 18.4 \pm 5.2 \text{ Mev},$$

$$H_8 = 2.21 \pm 0.15 \text{ Mev}.$$

The ratio of the two nonperturbative parameters is  $H_8/H_1 \approx 0.10$ , while it was determined to be 0.21 [4] if it was considered only to leading order in  $\alpha_s$ . Substituting  $H_1$  and  $H_8$  into (13) and (16) we can easily get the decay widths of  $\chi_{c0}$  and  $h_c$  into light hadrons

$$\Gamma(\chi_{c0} \rightarrow LH) = 12.4 \pm 3.2 \text{ Mev}, \quad (21)$$

$$\Gamma(h_c \rightarrow LH) = 0.71 \pm 0.07 \text{ Mev}. \quad (22)$$

Adding the radiative decay rate for  $\chi_{c0}$  whose branching fraction has been measured to be  $(0.66 \pm 0.18)\%$ , we obtain the total decay rate  $\Gamma(\chi_{c0}) = 12.5 \pm 3.2 \text{ Mev}$ , which agrees with the earlier Crystal Ball value  $14 \pm 5 \text{ Mev}$  [8], also with the new (preliminary) value of  $\Gamma_{tot}(\chi_{c0}) = 15.0_{-2.8}^{+3.2} \text{ Mev}$  measured by BES, using  $3.5 \times 10^6$   $\psi'(\psi(3686))$  events in  $\psi' \rightarrow \gamma\chi_{c0}$  and  $\chi_{c0} \rightarrow \pi^+\pi^-, K^+K^-$  decay channels [14]. The rate for decay  $h_c \rightarrow \gamma\eta_c$  has been estimated within a phenomenological framework to be about  $(340 - 380) \text{ kev}$  [13], and the total decay widths of  $h_c$  is then  $\Gamma(h_c) \approx 1.07 \text{ Mev}$ , which is also consistent with the experimental result [10].

For the electromagnetic decays to next to leading order in  $\alpha_s$ , we have

$$\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 6\pi e_c^4 \alpha^2 \left[1 + \left(\frac{\pi^2}{3} - \frac{28}{9}\right) \frac{\alpha_s}{\pi}\right] H_1, \quad (23)$$

$$\Gamma(\chi_{c2} \rightarrow \gamma\gamma) = \frac{8\pi}{5} e_c^4 \alpha^2 \left(1 - \frac{16\alpha_s}{3\pi}\right) H_1. \quad (24)$$

With the determined value for  $H_1$ , and  $e_c = 2/3$ ,  $\alpha = 1/137$ ,  $\alpha_s = 0.29$ , we predict

$$\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = (3.72 \pm 1.11) \text{ kev}, \quad (25)$$

which is in agreement with the observed value  $(4.0 \pm 2.8) \text{ kev}$  by Crystal Ball [8], and also predict

$$\Gamma(\chi_{c2} \rightarrow \gamma\gamma) = (0.49 \pm 0.15) \text{ kev}, \quad (26)$$

which is larger than the E760 value [11] but smaller than the CLEO value [12].

It is important to have a reasonable estimate for the theoretical uncertainties in our results. The two main sources of theoretical errors are relativistic corrections and higher-order perturbative corrections. Our formula (13)–(16) are only valid to leading order in  $v^2$  and high order relativistic corrections are not known at present. The error due to neglecting high order relativistic contributions could be of order  $v^2 \approx 30\%$ . On the other hand, we find that the one-loop coefficients in (13)–(16) are very large and strongly depend on the scale  $\mu$ . It is well known that when working to all order in  $\alpha_s$ , the decay rates, being the physical observables, will not rely on the choice of  $\mu$ . However we only do calculation to next-to-leading order in  $\alpha_s$ , therefore the analyses for the scale dependence of the results and the estimates for the higher-order effects are needed. The results are shown in Fig.2 and Fig.3 for decay rates  $\Gamma(\chi_{c0} \rightarrow LH)$  and  $\Gamma(h_c \rightarrow LH)$  respectively. For the running coupling constant  $\alpha_s$  with two-loops, three values  $\Lambda_{\overline{MS}}^{(3)} = 200 \text{ Gev}$ ,  $250 \text{ Gev}$ , and  $300 \text{ Gev}$  are used. The pictures show that our results are quite stable in the case of large  $\mu$ , say,  $\mu > 2m_c$ . In the physically motivated range  $\mu = m_c$  to  $2m_c$ , the decay rates vary from 15Mev to 9Mev for  $\Gamma(\chi_{c0} \rightarrow LH)$  and

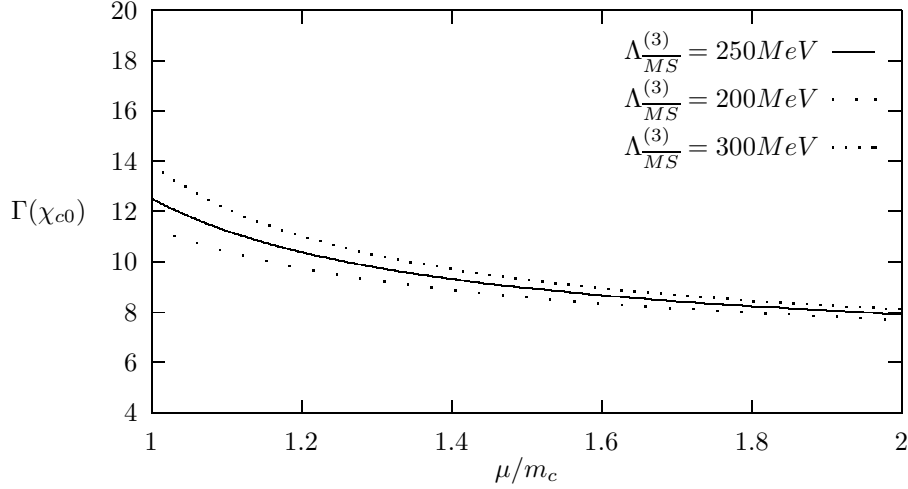


Figure 1: Renormalization scale dependence of the decay width  $\Gamma(\chi_{c0} \rightarrow LH)$

from 0.7MeV to 0.6MeV for  $\Gamma(h_c \rightarrow LH)$  respectively, while the obtained two phenomenological parameters  $H_1 = 22.0 - 19.0 \text{ MeV}$ ,  $H_8 = 2.3 - 3.1 \text{ MeV}$  and the ratio  $H_8/H_1 = 0.11 - 0.16$ . It is interesting to note that, although there are large theoretical uncertainties due to the scale dependence and higher-order QCD corrections, our estimate for  $\Gamma(\chi_{c0} \rightarrow LH)$  is enhanced greatly compared with the previous leading order result  $\Gamma_{tot}(\chi_{c0}) = (4.8 \pm 0.7) \text{ MeV}$  [4], which is smaller by a factor of three than the experimental value.

Some comments on the relativistic corrections might be in order. It is obviously difficult to perform a complete analysis for the  $O(v^2)$  corrections, because it must involve more higher dimensional four-fermion operators whose matrix elements are difficult to estimate at present. However, we might have some phenomenological analyses for the relativistic corrections in the color-singlet part. Just as the ratio  $\Gamma(\chi_{c0} \rightarrow 2\gamma)/\Gamma(\chi_{c2} \rightarrow 2\gamma)$ , which was discussed in ref.[15], the color-singlet contribution to  $\Gamma(\chi_{c0} \rightarrow 2g)/\Gamma(\chi_{c2} \rightarrow 2g)$  will receive relativistic corrections from two sources, i.e. the kinematic part and the dynamical part. In the language of the potential model, the color singlet matrix element  $H_1$  is proportional to  $R'_P(0)$ , the derivative of the wave function at the origin for the P-states. Due to a strong attractive spin-orbital force induced by one gluon exchange between quarks, which is also verified by the lattice calculations for the spin-dependent potentials between a heavy quark and an antiquark [16], the  $\chi_{c0}$  wave function in coordinate space will become narrower than the  $\chi_{c2}$  wave function in which the spin-orbital force is repulsive, and therefore the derivative of the wave function at the origin becomes larger for  $\chi_{c0}$  than that for  $\chi_{c2}$ . As a result, the dynamical relativistic effect will enhance  $H_1$  for  $\chi_{c0}$  relative to  $H_1$  for  $\chi_{c2}$ , and this effect is found to be dominant over the kinematic relativistic corrections [15]. This result might indicate that  $O(v^2)$  corrections may further make  $\Gamma(\chi_{c0} \rightarrow LH)$  enhanced. As for the relativistic corrections in the color-octet part, more



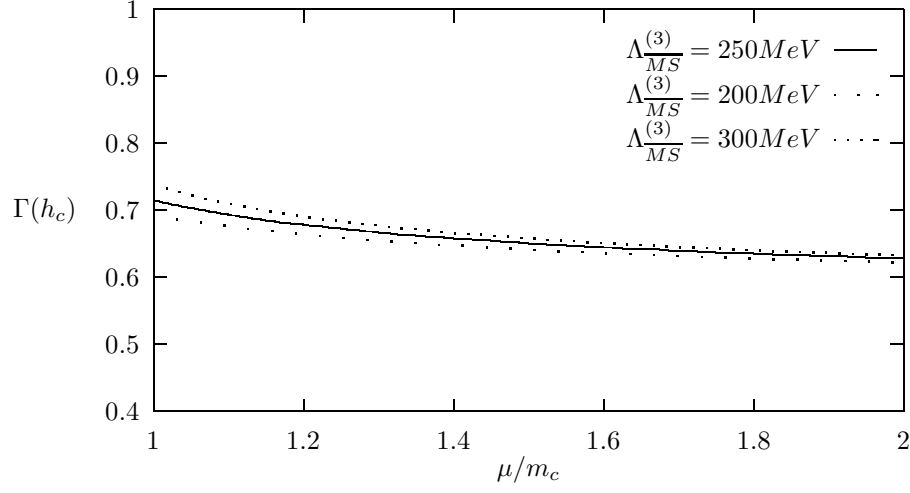


Figure 2: Renormalization scale dependence of the decay width  $\Gamma(h_c \rightarrow LH)$

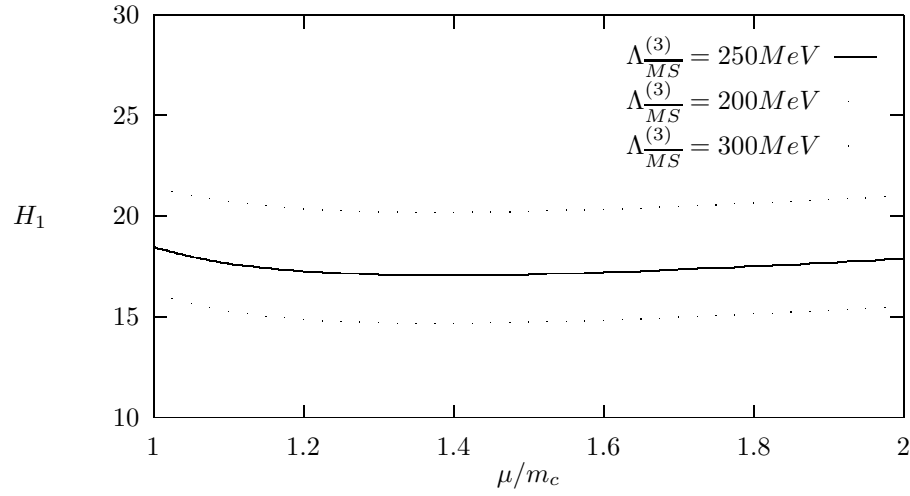


Figure 3: Renormalization scale dependence of  $H_1$

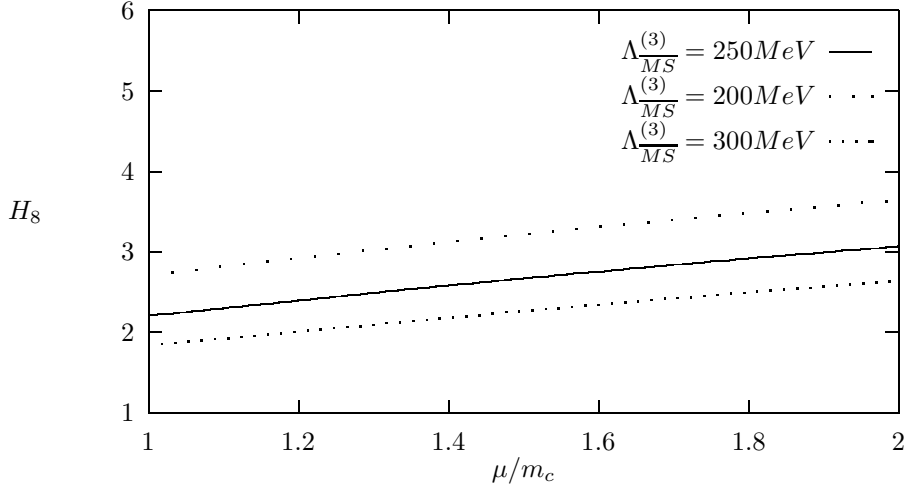


Figure 4: Renormalization scale dependence of  $H_8$

considerations are apparently needed in the future work.

In this paper we give the decay rates of four P-wave quarkonium states into light hadrons to leading order in  $v^2$  and next-to-leading order in  $\alpha_s$ . They are expressed in terms of two nonperturbative parameters  $H_1$  and  $H_8$ . Calculations in dimensional regularization scheme show that the infrared divergences, which appeared in the inclusive decay amplitudes for  $Q\bar{Q} \rightarrow q_l\bar{q}_l g$  and  $Q\bar{Q} \rightarrow ggg$ , can be cancelled explicitly by the contributions of color-octet operators in NRQCD. The finite coefficients of  $H_1$  and  $H_8$  are given to next to leading order in  $\alpha_s$ . Using the derived theoretical results and the measured decay widths of  $\chi_{c1}$  and  $\chi_{c2}$  we estimate  $H_1$ ,  $H_8$  and the decay widths of  $\chi_{c0}$  and  $h_c$ . The determined values are very different from the previous values obtained by neglecting the next-to-leading order QCD corrections [4]. In our results  $H_1$  is much larger than  $H_8$ , and the decay width of  $\chi_{c0}$  gets enhanced greatly due to  $O(\alpha_s)$  corrections. As a result, the predicted  $\chi_{c0}$  hadronic decay width and electromagnetic decay width both could be in agreement with or close to the data. These significant differences indicate that QCD radiative corrections are very important in understanding the decays of P-wave quarkonium. However, our results are valid only to leading order in  $v^2$  and next-to-leading order in  $\alpha_s$ . The large one-loop coefficients appearing in the expressions of the decay rates indicate that higher order QCD corrections may be important and that our results strongly depend on the choice of renormalization scale. More precise analyses must involve relativistic corrections and higher-order QCD corrections, which will include more matrix elements of higher dimensional operators.

We would like to thank Professor E. Braaten for pointing out a numerical error in (5) and (11) by comparing their recent result base on the threshold expansion method with our result by using the covariant projection method in dimensional regularization. It is turned out that the two methods in dimensional

regularization give identical results for the color-singlet sector of the P-wave decay widths, and are consistent with the previous calculation of Barbieri *et al.* using the binding energy as the infrared cutoff.

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